

GCE

Further Mathematics A

Y534/01: Discrete Mathematics

Advanced Subsidiary GCE

Mark Scheme for November 2020

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2020

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f”.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

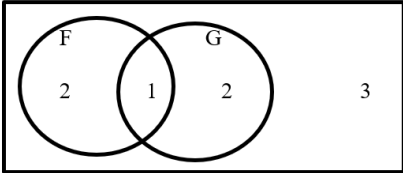
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Mark	Guidance																																																																																																								
1	(a)	<table border="1"> <tr><td>A</td><td>0</td><td></td><td></td><td></td><td>8</td><td></td><td></td><td></td><td>-1</td><td></td><td></td><td></td></tr> <tr><td>B</td><td></td><td>5</td><td></td><td></td><td></td><td>4</td><td></td><td></td><td></td><td>3</td><td></td><td></td></tr> <tr><td>C</td><td></td><td></td><td>1</td><td></td><td></td><td></td><td>1</td><td></td><td></td><td></td><td>1</td><td></td></tr> <tr><td>D</td><td></td><td></td><td></td><td>8</td><td></td><td></td><td></td><td>7</td><td></td><td></td><td></td><td>6</td></tr> </table> <table border="1"> <tr><td>A</td><td>7</td><td></td><td></td><td></td><td>-5</td><td></td><td></td><td></td><td>6</td><td></td><td></td><td></td></tr> <tr><td>B</td><td></td><td>2</td><td></td><td></td><td></td><td>1</td><td></td><td></td><td></td><td>0</td><td></td><td></td></tr> <tr><td>C</td><td></td><td></td><td>0</td><td></td><td></td><td></td><td>0</td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>D</td><td></td><td></td><td></td><td>2</td><td></td><td></td><td></td><td>1</td><td></td><td></td><td></td><td></td></tr> </table> <p>Output = 6</p>	A	0				8				-1				B		5				4				3			C			1				1				1		D				8				7				6	A	7				-5				6				B		2				1				0			C			0				0						D				2				1					<p>B1 B1</p> <p>M1 A1</p> <p>B1 [5]</p>	<p>Allow compressed rows or tables used differently</p> <p>Values of <i>B</i> are 5, 4, 3, 2, 1, 0 Values of <i>C</i> are 1, 1, 1, 0, 0 Ignore if there is an extra 0 at the end</p> <p>Their first two <i>D</i> values = $B + 3C$ All <i>D</i> values correct: 8, 7, 6, 2, 1 Ignore if there is an extra 0 at the end</p> <p>6</p>
		A	0				8				-1																																																																																																	
B		5				4				3																																																																																																		
C			1				1				1																																																																																																	
D				8				7				6																																																																																																
A	7				-5				6																																																																																																			
B		2				1				0																																																																																																		
C			0				0																																																																																																					
D				2				1																																																																																																				
<p>Value of <i>B</i> is reduced by 1 each time that STEP 6 is used until it reaches 0, when STEP 7 and STEP 8 mean that the algorithm stops</p>	<p>B1 B1</p> <p>[2]</p>	<p><i>B</i> reduces by 1 in each pass (NOT just <i>B</i> is a counter) <i>B</i> reaches 0 or referring to STEP 7 and/or STEP 8 or STOP instruction</p>																																																																																																										
1	(c)	For any given input the output is always the same.	<p>B1</p> <p>[1]</p>	The output is a function of the input only																																																																																																								
1	(d)	<i>B</i> reduces to 0 after <i>n</i> passes through STEP 6, so run time is <i>kn</i> which is a linear function of <i>n</i> Hence efficiency is $O(n)$ (as given in question)	<p>B1</p> <p>[1]</p>	Identifying <i>n</i> passes (allow $n \pm 1$) Or ' <i>B</i> reduces by 1 in each pass' so (run time) is linear, or equivalent																																																																																																								
1	(e)	For large enough <i>n</i> , the run time of X increases more rapidly than the run-time of Y.	B1	Referring to 'for large values of <i>n</i> ' (or <i>B</i>) and not just saying that the number of digits is smaller than <i>n</i> . Not just specific examples.																																																																																																								
		<p>Alternative method The number of digits in <i>B</i> is $1 + \text{INT}(\log_{10} B)$.</p> <p>For any positive values of <i>k</i> and <i>m</i> there will be a value <i>N</i> for which the run-time of X > the run time of Y for all $n > N$.</p>	B1	Any reasonable attempt at number of digits in <i>B</i> that involves logarithms																																																																																																								
			[1]	A level candidates may give $O(\log n) \subset O(n)$ Need not explain this																																																																																																								

Question		Answer	Mark	Guidance										
2	(a)	How can I fit the packages in the boxes? She needs to find a solution, although it may not be a good solution She does not need to enumerate the number of solutions	B1 [1]	Correct question chosen with a valid reason										
2	(b)	A solution may not exist because the packages may be the wrong shapes. May have a package that is too big ($> 0.25 \text{ m}^3$) Could have packages that cannot be paired together to fit in six boxes (e.g. $6 \times 0.15 \text{ m}^3$ and $4 \times 0.025 \text{ m}^3$)	B1 [1]	Shape matters as well as volume Or explaining why total volume = 1 m^3 does not mean that individual volumes are small enough										
2	(c)	Would need all four boxes full, with no space left A(0.20), C(0.15), D(0.25) must be in 3 separate boxes. I(0.12) and J(0.12) cannot be in the same box as A, C or D so I and J are together in another box The box with I and J has 0.01 spare, but the smallest package has volume 0.02. So impossible to fill 4 boxes	B1 B1 [2]	This first statement may be implied I and J must be together (e.g showing a solution in which A, C and D are in separate boxes <u>and</u> I, J are together in a box) Explaining why this means that 4 boxes are not enough (not just showing that first fit decreasing fails)										
2	(d)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Box 1</td> <td>A B</td> </tr> <tr> <td>Box 2</td> <td>C E F G</td> </tr> <tr> <td>Box 3</td> <td>D</td> </tr> <tr> <td>Box 4</td> <td>H I</td> </tr> <tr> <td>Box 5</td> <td>J</td> </tr> </table>	Box 1	A B	Box 2	C E F G	Box 3	D	Box 4	H I	Box 5	J	M1 A1 [2]	A, B, C, D placed correctly (if letters are not used, 0.20, 0.05, 0.15, 0.25 placed correctly = M1 A0) All correct, using letters
Box 1	A B													
Box 2	C E F G													
Box 3	D													
Box 4	H I													
Box 5	J													
2	(e)	$8 - (3 + 3) + 1$ $= 3$	M1 A1 [2]	Or $3 + 3 - 1 = 5$ shown on a Venn diagram 3, from correct working seen 										

Question			Answer	Mark	Guidance																																																																															
3	(a)	(i)	e.g. A – B – A – D – H e.g. A – B – C – D – E – C – D – H e.g. A – D – E – D – H	B1 [1]	A walk from A to H that repeats an arc (or arcs)																																																																															
		(ii)	e.g. A – D – C – E – D – H e.g. A – B – C – D – A – G – H	B1 [1]	A trail from A to H (does not repeat any arcs) that does repeat a node (or nodes)																																																																															
		(iii)	e.g. A – D – H e.g. A – B – C – D – E – G – H	B1 [1]	A path from A to H (does not repeat any nodes)																																																																															
3	(b)	<table border="1" style="margin-bottom: 10px;"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> </tr> </thead> <tbody> <tr> <th>A</th> <td>-</td> <td>3</td> <td>-</td> <td>8</td> <td>-</td> <td>-</td> <td>x</td> <td>-</td> </tr> <tr> <th>B</th> <td>3</td> <td>-</td> <td>5</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> </tr> <tr> <th>C</th> <td>-</td> <td>5</td> <td>-</td> <td>4</td> <td>3</td> <td>-</td> <td>-</td> <td>-</td> </tr> <tr> <th>D</th> <td>8</td> <td>-</td> <td>4</td> <td>-</td> <td>5</td> <td>-</td> <td>-</td> <td>9</td> </tr> <tr> <th>E</th> <td>-</td> <td>-</td> <td>3</td> <td>5</td> <td>-</td> <td>7</td> <td>6</td> <td>-</td> </tr> <tr> <th>F</th> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>7</td> <td>-</td> <td>-</td> <td>-</td> </tr> <tr> <th>G</th> <td>x</td> <td>-</td> <td>-</td> <td>-</td> <td>6</td> <td>-</td> <td>-</td> <td>2</td> </tr> <tr> <th>H</th> <td>-</td> <td>-</td> <td>-</td> <td>9</td> <td>-</td> <td>-</td> <td>2</td> <td>-</td> </tr> </tbody> </table> <p> AB = 3 BC = 5 CE = 3 CD = 4 EG = 6 GH = 2 EF = 7 </p>		A	B	C	D	E	F	G	H	A	-	3	-	8	-	-	x	-	B	3	-	5	-	-	-	-	-	C	-	5	-	4	3	-	-	-	D	8	-	4	-	5	-	-	9	E	-	-	3	5	-	7	6	-	F	-	-	-	-	7	-	-	-	G	x	-	-	-	6	-	-	2	H	-	-	-	9	-	-	2	-	<p>For reference</p> <p>B1 Choosing the 7 circled entries, shown on table Must be on the table and all correct (not any transposes)</p> <p>M1 A spanning tree for the 8 nodes need not match the arcs in the network, need not be MST</p> <p>A1 Correct tree drawn or correct arcs listed (not just on table)</p> <p>[3]</p>
	A	B	C	D	E	F	G	H																																																																												
A	-	3	-	8	-	-	x	-																																																																												
B	3	-	5	-	-	-	-	-																																																																												
C	-	5	-	4	3	-	-	-																																																																												
D	8	-	4	-	5	-	-	9																																																																												
E	-	-	3	5	-	7	6	-																																																																												
F	-	-	-	-	7	-	-	-																																																																												
G	x	-	-	-	6	-	-	2																																																																												
H	-	-	-	9	-	-	2	-																																																																												
	(c)	(i)	{x: 0 ≤ x ≤ 6}	B1 [1]	$x \leq 6, x \leq$ (their largest weight used to A or G) Allow strict inequality																																																																															
		(ii)	EG	B1 [1]	Follow through their spanning tree from (b)																																																																															

Question		Answer	Mark	Guidance
4	(a)	<p>Minimum project completion time = 60 days</p> <p>Critical activities: A, B, C, G, H, I, K</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>H, I, J, K added appropriately, apart from possibly not using a dummy (or using too many dummies) and possibly not showing arcs as directed. Durations need not be shown.</p> <p>Network correct, including the use of exactly one dummy (before or after I or J) and all arcs directed</p> <p>Follow through their network, even if only A to G</p> <p>Forward pass, or implied from minimum time correct 60</p> <p>Backward pass, or implied from critical activities correct A, B, C, G, H, I, K (and no others)</p>
4	(b)	<p>The start of an activity may be delayed (e.g. a delay in the delivery would hold up the start of G or an activity may take longer than Bob has planned)</p> <p>There may not be enough workers available who can do the activities at the required times</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>A reason why the duration of an activity may be extended</p> <p>A resourcing issue related to the worker availability</p>
4	(c)	<p>F has the longest float = $25 - 1 - 5 = 19$ days</p> <p>The clearing of the work area can start at any time from the end of A (day 1) to day 20 without delaying the project</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>19 or ft their forward and backward passes</p> <p>Interpretation in context, must refer to 'clearing work area' and start or finish times (or needing 5 days from the window of 24 days) or reference to opportunity window</p>

Question		Answer	Mark	Guidance
5	(a)	5	B1 [1]	cao
5	(b)	<p>Row minima: $A = \min\{2k, -2, 1 - k\}$ $B = -5$ $C = -4$ $D = -5$ $2k = -4$ when $k = -2$ $1 - k = -4$ when $k = 5$</p> <p>So A is a play-safe strategy when $-2 \leq k \leq 5$</p>	<p>M1 $2k, -2, 1 - k$ (may also include 4)</p> <p>A1 -4 seen or used appropriately</p> <p>B1 Critical value $k = -2$ (or as an inequality)</p> <p>B1 Critical value $k = 5$ (or as an inequality)</p> <p>M1 Appropriate sketch graph showing $y = 2k, y = 1 - k$ and at least one of $y = -2, y = -4$</p> <p>Or implied from answer</p> <p>A1 Accept $<$ instead of (either or both) \leq</p> <p>[6]</p>	
5	(c)	<p>E: $\max\{2k, 4\}$, F: 4 G: $\max\{4, 1 - k\}$, H: 6 $\max\{2k, 4\} \geq 4$ and $\max\{4, 1 - k\} \geq 4$ and F = 4 Hence col minimax = 4</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Any three of these correct (not implied), 'max' may be implied</p> <p>4 from correct working seen</p>

Question		Answer	Mark	Guidance
6	(a)	$6 - x - y$	B1 [1]	This expression, in any form
6	(b)	(i)	M1 M1 A1 [3]	$5x + 3y$ + $2(6 - x - y)$ or 2(their expression from part (a)) $3x + y + 12$ seen or explaining why constant can be ignored
		(ii)	M1 A1 [2]	$y \geq 6 - x - y$ $\Rightarrow x + 2y \geq 6$ $y \geq$ their expression from part (a) Leading to given expression
		(iii)	M1 A1 [2]	$x \leq 3.5$ and $x + 2y \geq 6$ $\Rightarrow y \geq 1.25$ (or $y \geq 0$) x is at most 3.5 (or x is at most 5) and $x + 2y$ is at least 6 Leading to implication that y cannot be negative (as given) Argued algebraically or in words

Question	Answer	Mark	Guidance															
<p>6 (c)</p>	<table border="1" data-bbox="387 1007 1113 1114"> <tr> <td>x</td> <td>0</td> <td>3.5</td> <td>3.5</td> <td>0</td> </tr> <tr> <td>y</td> <td>3</td> <td>1.25</td> <td>1.5</td> <td>5</td> </tr> <tr> <td>$3x+y$</td> <td>3</td> <td>11.75</td> <td>12</td> <td>5</td> </tr> </table> <p>Tamsin should spend 3.5 hours walking the coast path 1.5 hours visiting the bird sanctuary</p> <p>1 hour visiting the garden centre</p>	x	0	3.5	3.5	0	y	3	1.25	1.5	5	$3x+y$	3	11.75	12	5	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>May have other lines shown, e.g. profit lines</p> <p>Assume x is on the horizontal axis and y on the vertical axis, unless labelled to the contrary</p> <p>Vertical boundary $x = 3.5$</p> <p>Line $x + 2y = 6$ through $(0, 3)$ and $(4, 1)$</p> <p>Line $x + y = 5$, through $(0, 5)$ and $(4, 1)$</p> <p>Shading to show this feasible region</p> <p>Checking vertices or using a sliding profit line May be implied from their solution provided consistent with their feasible region</p> <p>Interpretation of their x and y in context, provided $x \leq 3.5$ and both are ≥ 0</p> <p>$(6 - \text{their } x - \text{their } y)$ hours (≥ 0) visiting the garden centre</p>
x	0	3.5	3.5	0														
y	3	1.25	1.5	5														
$3x+y$	3	11.75	12	5														

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored